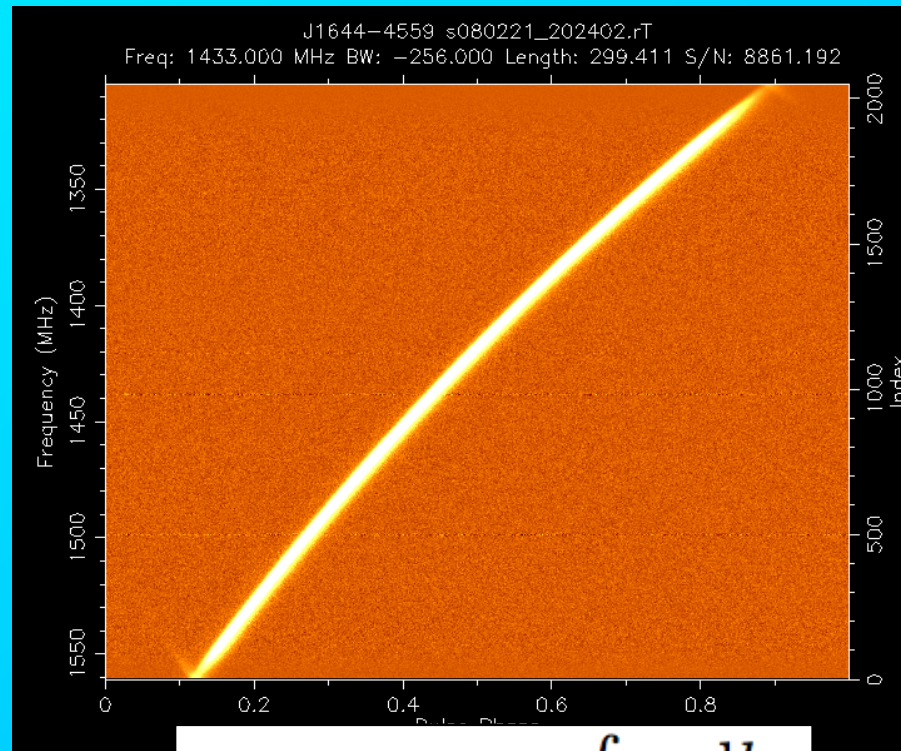


# Dispersion Variations: Impact and Analysis

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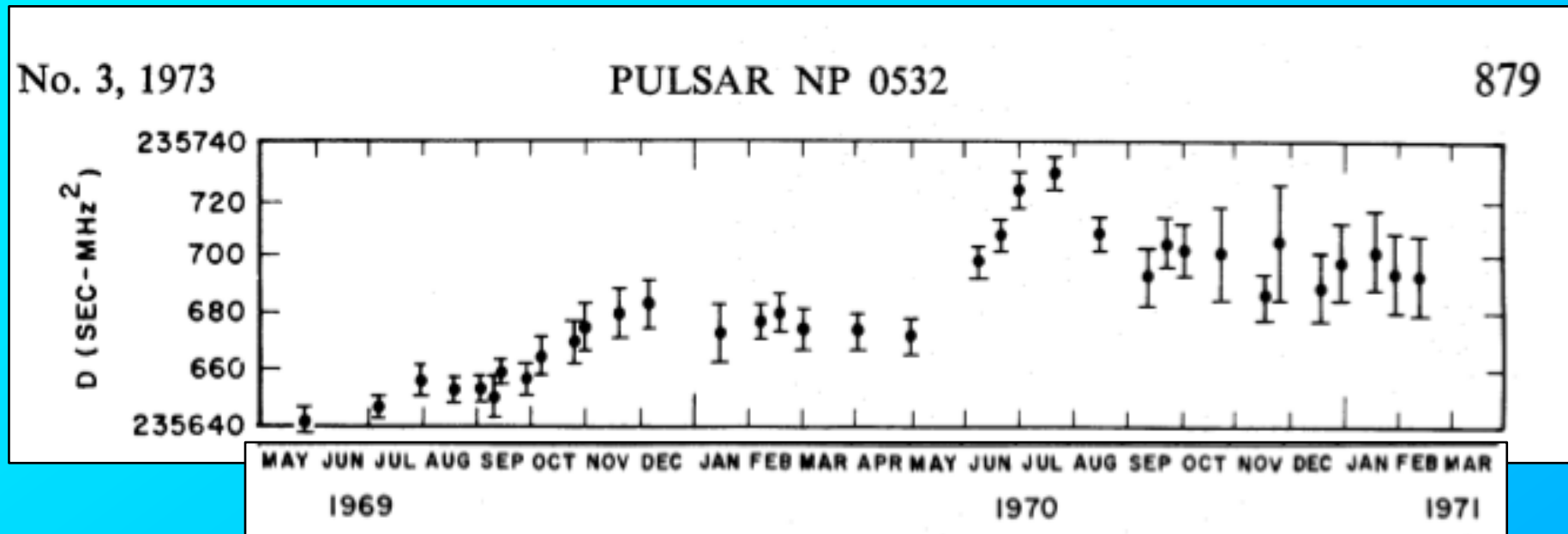


$$\Delta t_{\text{DM}} \sim \frac{\int n_e dl}{\nu^2}$$



# Dispersion Variations

- Variations first detected in Crab and Vela pulsars
  - Rankin & Counselman (1973)
  - Hamilton, Hall & Costa (1985)



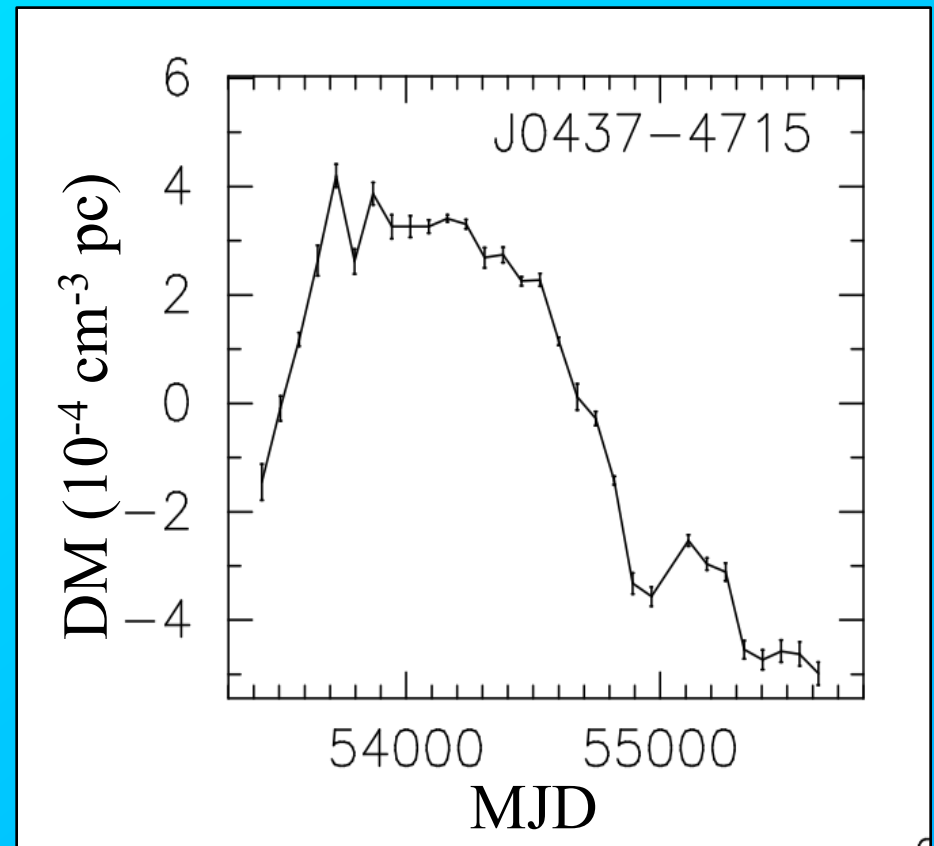
For Vela, rate  $\sim 0.04 \text{ cm}^{-3} \text{ pc yr}^{-1}$

- Uncorrected DM variations add noise to timing data
- Spectrum is red but often contains significant power at frequencies  $\sim 1 \text{ yr}^{-1}$
- Uncorrected DM variations will bias fitted pulsar parameters, e.g. parallax
- Will also contribute power to unmodelled signals, e.g. from gravitational waves

$$\Delta\text{DM} = 10^{-4} \text{ cm}^{-3} \text{ pc}$$



$$\Delta t_{\text{DM}} = 212 \text{ ns at } 1.4 \text{ GHz}$$



(Keith et al. 2013)

# DM Correction

- Observed ToAs are sum of frequency-independent “common-mode” terms  $t_{\text{CM}}$  (e.g., clock errors, GW, etc) and interstellar delays  $t_{\text{DM}}$  – assume  $\sim \lambda^2$

$$t_{\text{OBS}} = t_{\text{CM}} + t_{\text{DM}}(\lambda/\lambda_{\text{REF}})^2$$

- The interstellar term  $t_{\text{DM}}$  is noise – want to minimise it
  - Observe at  $\sim$ zero wavelength, i.e., X-ray or  $\gamma$ -ray
  - Observe at two or more wavelengths,  $\lambda_1$  and  $\lambda_2$  (with  $\lambda_1 > \lambda_2$ )
- Can then solve for  $t_{\text{DM}}$  and  $t_{\text{CM}}$ :

$$\tilde{t}_{\text{DM}} = (t_{\text{OBS},1} - t_{\text{OBS},2})\lambda_{\text{REF}}^2 / (\lambda_1^2 - \lambda_2^2),$$

$$\tilde{t}_{\text{CM}} = (t_{\text{OBS},2}\lambda_1^2 - t_{\text{OBS},1}\lambda_2^2) / (\lambda_1^2 - \lambda_2^2).$$

- $t_{\text{DM}}$  is proportional to the DM variation
- $t_{\text{CM}}$  is what we really want!
- We need to minimise the uncertainty in  $t_{\text{CM}}$ :

$$\sigma_{t_{\text{CM}}} = \left[ \sigma_{t_{\text{OBS},2}}^2 + \left( \frac{\lambda_2^2}{\lambda_1^2} \sigma_{t_{\text{OBS},1}} \right)^2 \right]^{1/2} \left( \frac{\lambda_1^2}{\lambda_1^2 - \lambda_2^2} \right)$$

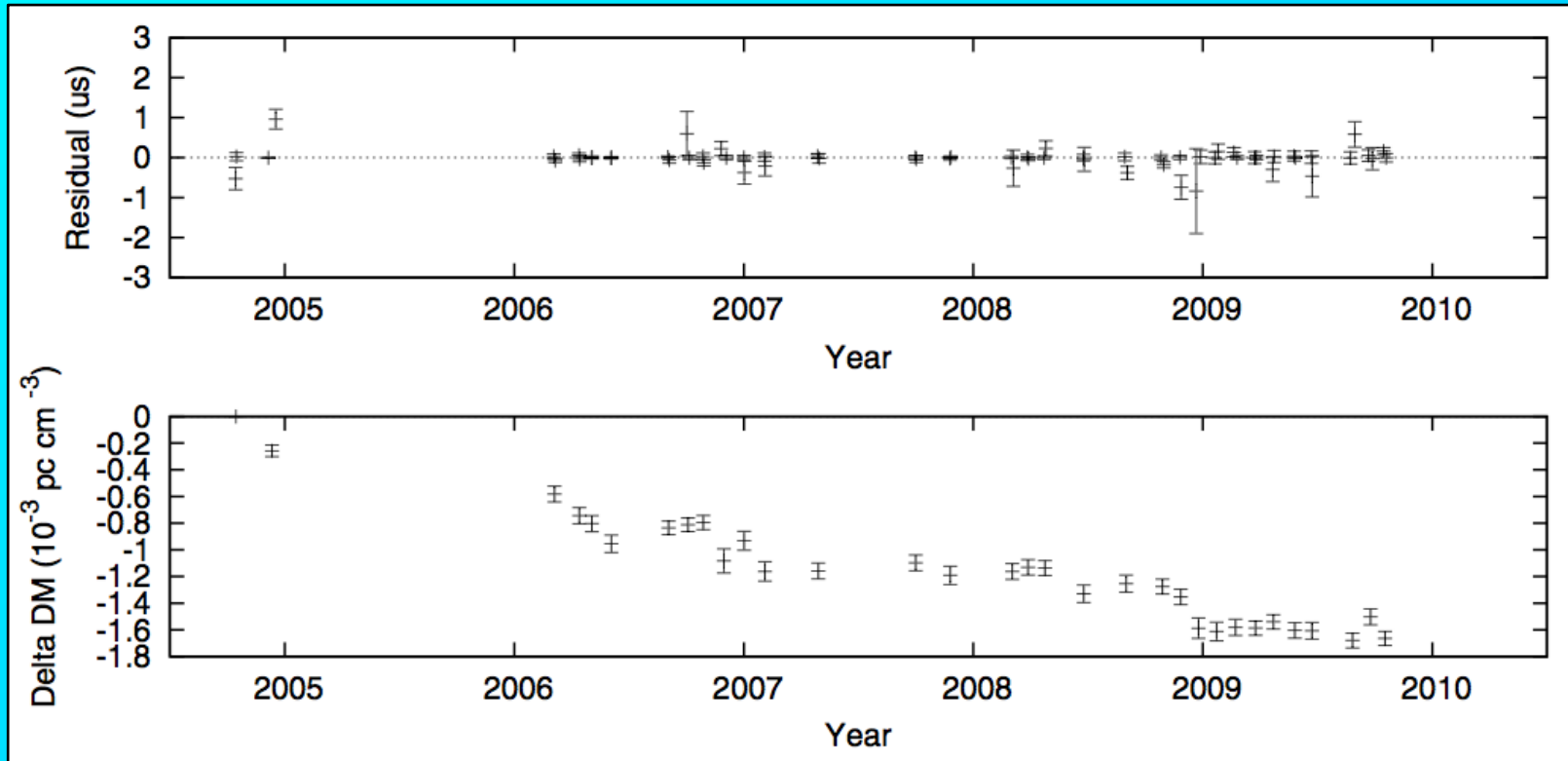
i.e., need  $\lambda_2 \ll \lambda_1$  and small  $\sigma_t$

– note that  $\sigma_{t_1}$  can be larger than  $\sigma_{t_2}$  by a factor  $\sim (\lambda_1/\lambda_2)^2$

- Sum of DM corrections is constrained to zero
- Also, need to ensure that  $t_{\text{CM}}$  is not covariant with timing model terms, e.g.,  $\mathbf{v}$ ,  $\dot{\mathbf{v}}$ , etc.

# DM Variations - PSR J1909-3744

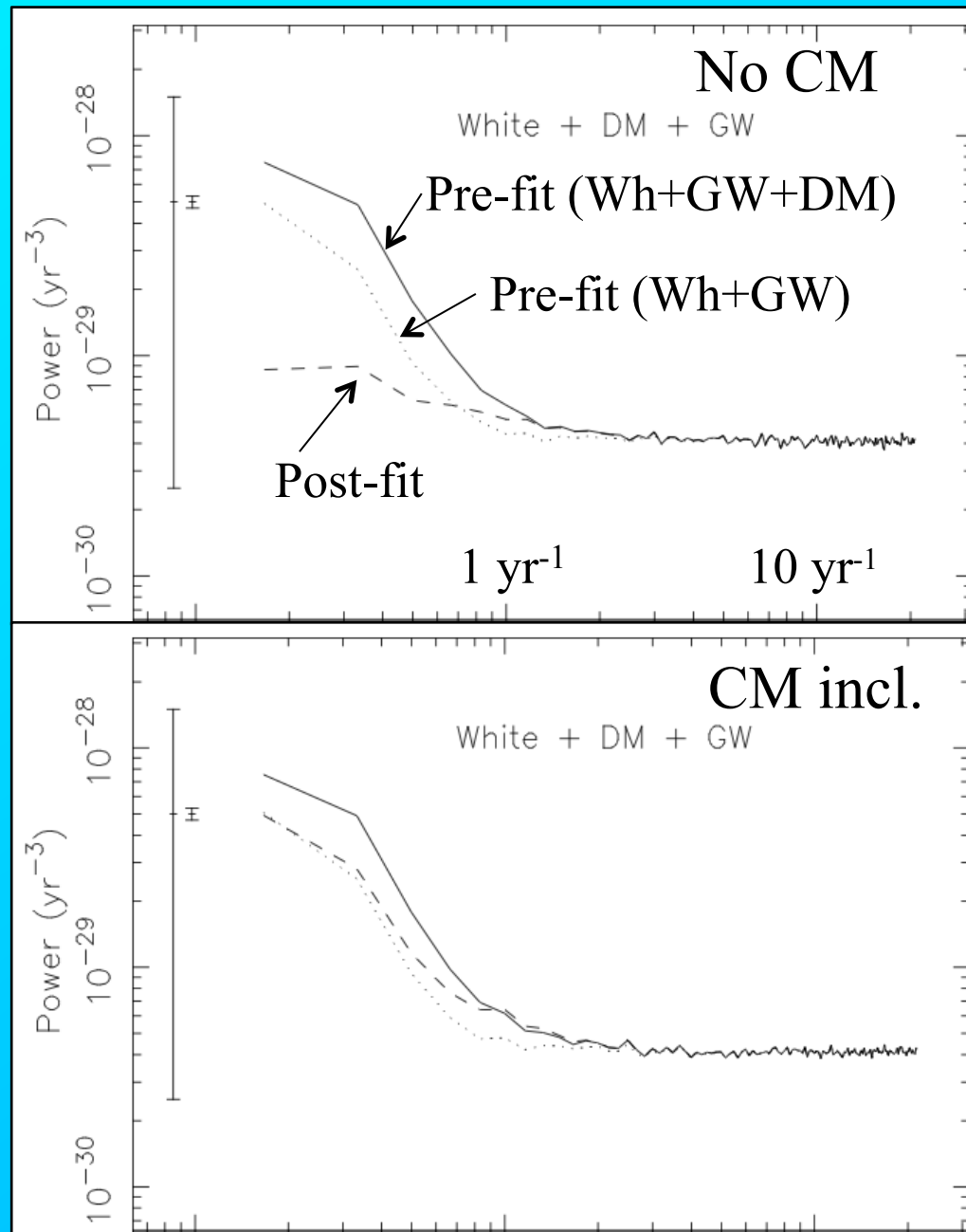
## NANOGrav



- GBT: 820 and 1400 MHz
- $\Delta\text{DM}$  from dual-band observations within 15-day span  
(Demorest et al. 2013)

# Effect of CM Term

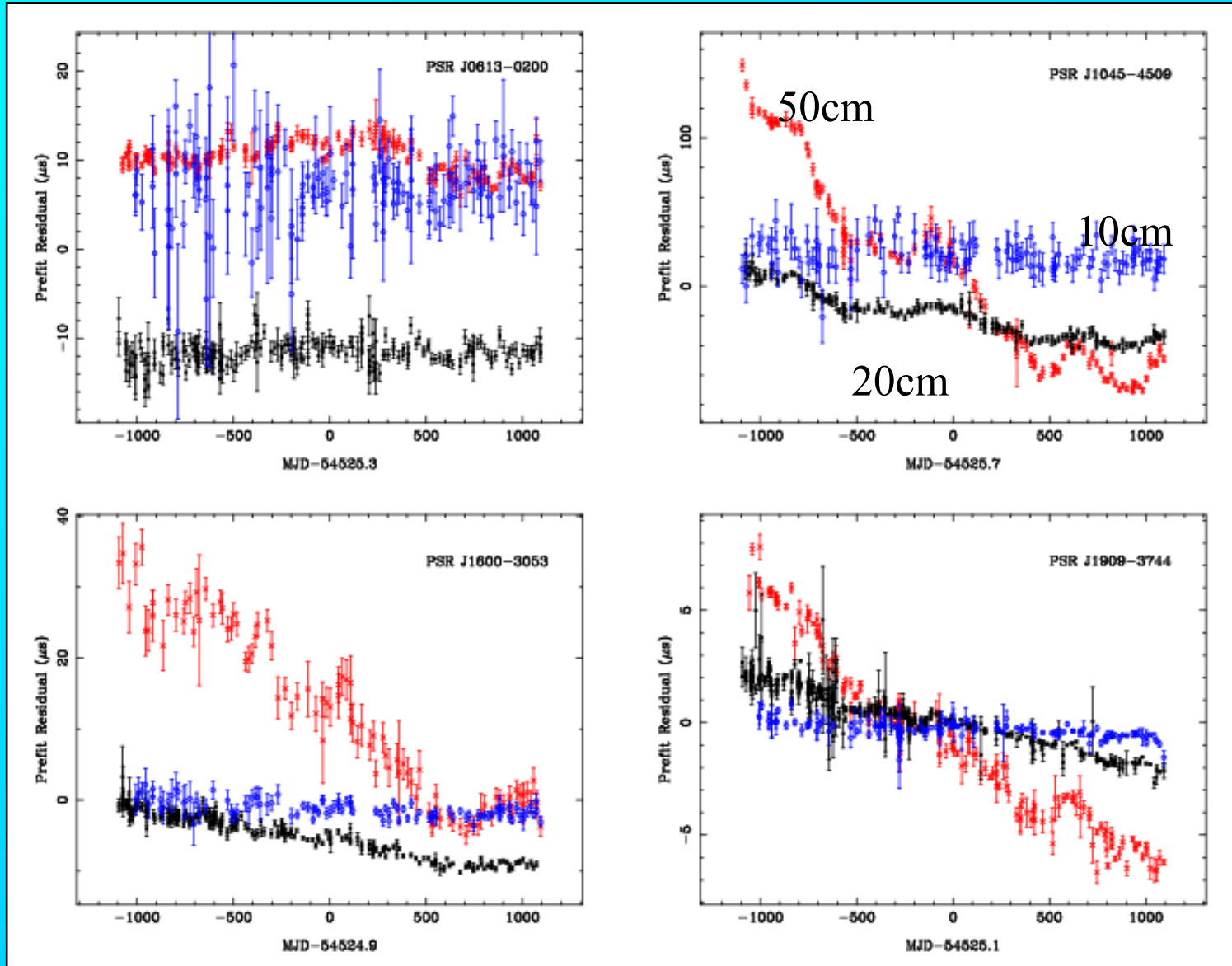
- If CM term not included in fit, power is extracted from freq-independent variations and coupled into DM variations
- With CM term included, all freq-independent power (e.g., GW signal, clock errors) is contained in CM values  
(Keith et al. 2013)



Power spectra of timing residuals



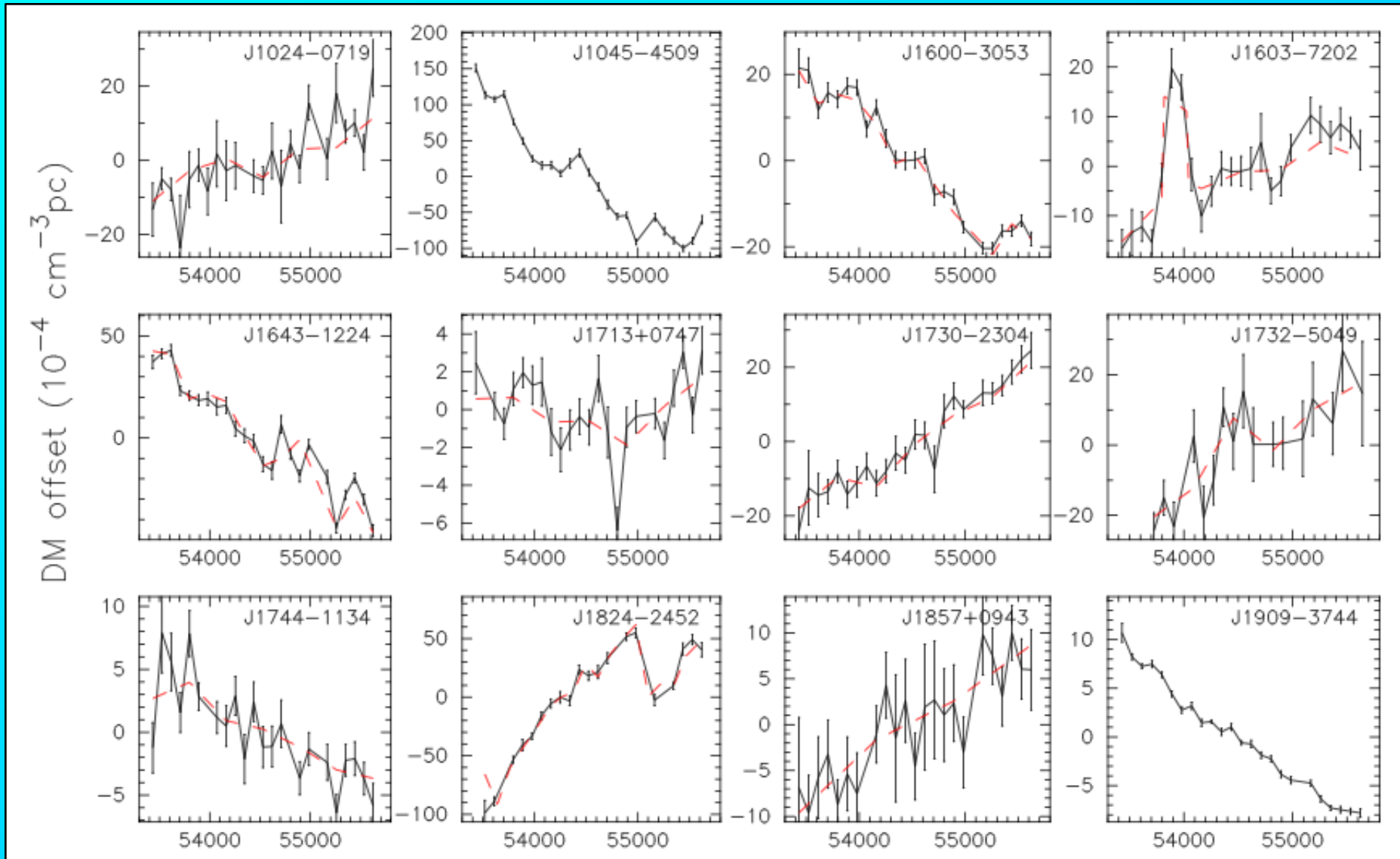
# PPTA Three-Band Timing Residuals



(Manchester et al. 2013)



# DM Variations for PPTA pulsars



(Keith et al. 2013)

# DM Smoothing

- For PPTA, data are better sampled – obs every 2-3 weeks
- Benefit in averaging multi-band data over a longer span – reduces effective  $\sigma_t$
- Uniform sampling at  $T_s$  with linear interpolation
- $T_s = 1/f_c$ , where  $f_c$  is frequency where DM-fluctuation and white-noise powers are equal
- Little effect of sensitivity of GW detection since expected  $A^2_{\text{GW}} \sim f^{-13/3}$  – steeper than spectrum of DM fluctuations
- Improves precision of model terms with spectral power at  $f \sim f_c$ , e.g., parallax

# Effect of DM Corrections: PPTA Psrs

- Nine psrs where DM correction clearly beneficial
- For six more, some benefit
- For others, added noise outweighs benefit or non- $v^2$  variations

PSR	$\frac{ \Delta\nu }{\sigma_\nu}$	$\frac{ \Delta\dot{\nu} }{\sigma_{\dot{\nu}}}$	$\frac{\Sigma_{\text{post}}}{\Sigma_{\text{pre}}}$	$\frac{\bar{P}_{\text{post}}}{P_{\text{pre}}}$	Imp.
J0437–4715	92	48	0.6	0.15-0.25	Y
J0613–0200	0.16	2.9	1.1	0.3-1.2	y
J0711–6830	3.9	5.5	1.0	0.4-1.6	y
J1022+1001	1.4	0.3	1.0	0.6-2.6	n
J1024–0719	1	0.91	1.0	0.2-0.7	Y
J1045–4509	28	11	0.7	0.22-0.39	Y
J1600–3053	35	0.51	1.0	0.4-0.8	Y
J1603–7202	2.4	2.5	1.0	0.2-0.9	Y
J1643–1224	11	0.73	1.7	1.3-3.1	N
J1713+0747	3.2	6.2	1.0	0.2-0.7	Y
J1730–2304	6.5	1.8	1.1	0.9-3.2	n
J1732–5049	2.6	2.8	1.0	0.4-1.4	y
J1744–1134	5.4	0.48	1.0	0.5-2.0	n
J1824–2452A	24	31	0.7	0.29-0.56	Y
J1857+0943	4.3	1	1.0	0.2-1.0	y
J1909–3744	28	5	1.0	0.44-0.79	Y
J1939+2134	13	1.7	0.7	0.34-0.67	Y
J2124–3358	0.25	0.056	1.0	0.5-1.9	y
J2129–5721	3	2.1	1.1	0.7-2.8	n
J2145–0750	0.22	0.18	1.0	0.2-1.0	y

(Keith et al. 2013)

# Scattering and non- $v^2$ Delays

- Scattering delays scale roughly as  $\lambda^4$
- If not separately solved for, will bias DM corrections and contribute excess noise to high-frequency ToAs
- Observed in PSR J1939+2134 and J1643-1224
- For J1643-1224, DM correction increases the white timing noise at 20cm by more than a factor of three
- Will limit use of very low frequencies for DM correction unless scattering delay is separately measured



# Summary

- DM variations are a major contributor to timing noise for MSPs at GHz frequencies
- Best solution is to avoid them by timing X-ray,  $\gamma$ -ray, or at least relatively high radio frequencies  $>\sim 3$  GHz
- Observations at frequencies  $<\sim 2$  GHz must be corrected using observations at lower frequencies
- Frequencies  $<\sim 400$  MHz problematic for all but low-DM pulsars because scattering delays are starting to dominate
- With current systems, timing precision is generally limited by precision and accuracy of DM corrections – main motivation for development of ultra-wide-band receivers